*;;;?;;Analysis of Quantum Evaporation Process of Black Holes
in the Model of Expansive Nondecelerative Universe

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Abstract. In the model of Expansive Nondecelerative Universe, black hole
cannot totally evaporate via quantum evaporation process proposed by Hawk-
ing. In a limiting case, an equilibrium of gravitation creation and black hole
evaporation can be reached keeping the surface of its horizon
constant. This conclusion is in accordance with the second law of thermo-
dynamics.

1. Introduction

One of the greatest achievements of cosmology and astrophysics was for-
mulation of hypothesis on the existence of black holes followed by its undirect
experimental verification. Several problems relating to black holes have re-
mained still open, one of them concerns ways of decreasing their mass. Based
on quantum mechanics and thermodynamics Hawking suggested a solution of
the problem in the form of quantum evaporation. His theory has led, however,
to possibility of the total evaporation of black holes, a phenomenon that has
never been observed. In our previous contribution [1] we documented that a
decrease in the mass of a black hole via its evaporation contradicts the sec-
ond law of thermodynamics. In this contribution more details are given and,
in addition, independent modes of an evidence on the improbability of a black
hole mass decreasing, based on mutual consistency of the calculations treated
the gravitational field energy quantum, output and density, E_g, P_g and ϵ_g are
offered.

2. Theoretical background

In the model of Expansive Nondecelerative Universe (ENU) [1 - 3], the gauge
factor a , the cosmological time t_c and the mass of the Universe M_U are related
as follows

$$a = c.t_c = \frac{2G.M_U}{c^2} \qquad (1)$$

Due to the matter creation, Vaidya metrics [4] is applied in ENU. This mode
of treatment allows to localize the energy of the gravitational field. Density of

1

the gravitational field energy ϵ_g of a body with the mass m in the distance r is defined by relation

$$\epsilon_g = -\frac{R.c^4}{8\pi.G} = -\frac{3m.c^2}{4\pi.a.r^2}$$

(2)

where R is the scalar curvature. Contrary to a more frequently used Schwarzschild metrics (in which $\epsilon_g = 0$ outside a body, and $R = 0$), in Vaidya metrics $R \neq 0$ and ϵ_g may thus be quantified and localized also outside a body. For the energy of a quantum of the gravitational field, equation (3) was derived in [1]

$$|E_g| = \left(\frac{m.\hbar^3.c^5}{a.r^2}\right)^{1/4}$$

(3)

Gravitational output, i.e. the amount of the gravitational energy emitted by a body with the mass m within a time unit is defined as

$$P_g = \frac{d}{dt}\int \epsilon_g.dV \cong -\frac{m.c^3}{a} = -\frac{m.c^2}{t_c}$$

(4)

Hawking [5] rationalized the process of quantum evaporation of a black body; within the process a black body with the diameter r_{BH} evaporates photons with the energy

$$E_{BH} = \frac{\hbar.c}{r_{BH}}$$

(5)

The output of a black hole evaporation was expressed by Hawking as

$$P_{BH} = \frac{\hbar.c^2}{r_{BH}^2}$$

(6)

According to Hawking, a black hole with the mass m_{BH} will totally evaporate during the time

$$t \approx \frac{G^2.m_{BH}^3}{\hbar.c^4}$$

(7)

If the time t is substituted by cosmological time t_c being

$$t_c \cong 4.5x10^{17}s$$

(8)

then black holes completing their evaporation at present should have the initial mass

$$m_{BH}^o \approx 10^{12}kg$$

(9)

It was admitted by Hawking himself that in spite of a great efford, no such an evaporation was experimentally observed.

3. Evaporation of black holes analysed from the viewpoint of ENU

In the ENU model [1], the mutually related creation of matter and of gravitational energy simultaneously occurs. The laws of energy conservation still hold since the energy of gravitational field is negative and, consequently, the total energy of the Universe is thus exactly equal to zero [2, 5]. In case of black holes, the matter creation and evaporation are competitive opposed processes. Since a magnitude of the surface of a black hole horizon is proportional to entropy and the second law of thermodynamics may not be violated, during the matter creation and evaporation the surface of black hole horizon must not decrease. In a limiting case, when amounts of the created gravitation and evaporated matter are just balanced, the surface of the black hole horizon is constant. For such a case three postulates can be formulated:
   a)the energy of gravitational field quanta is identical to the energy of
   photons emitted at evaporation,
   b) gravitational output equals to output of the evaporation,
   c)density of the energy of black hole radiation is equal to the density of
gravitational energy.
   Justification of the above postulates is verified below.
   a)It follows from (3) and (5) that

$$\left(\frac{m_{BH}.\hbar^3.c^5}{a.r_{BH}^2}\right)^{1/4} \cong \frac{\hbar.c}{r_{BH}} \tag{10}$$

Using (1), Hawking relation (7) is directly obtained from (10) providing
that time t represents the cosmological time t_c.
From the viewpoint of ENU, relation (10) represents a limiting (i.e. the
lightest) black hole in a given cosmological time. Such a limiting black
hole may exist in cases when requirements on the equilibrium of creation
and evaporation, and those stemming from the second law of thermodynamics
are met.
   b) At the same time it follows from relations (4) and (6) that (in absolute
values)

$$\frac{m_{BH}.c^2}{t_c} \cong \frac{\hbar.c^2}{r_{BH}^2}$$
$$\tag{11}$$

Based on (11), Hawking relation (7) can again be derived, its interpretation
being identical to that offered in a).
   c) Using the Stefan-Boltzmann law and relation (2), it must hold in the
mentioned limiting case

$$\frac{3m_{BH}.c^2}{4\pi.a.r_{BH}^2} \cong \frac{4\sigma.T^4}{c}$$
$$\tag{12}$$

3

Relation (12) can be simplified using formulas

$$m_{BH} = \frac{r_{BH}.c^2}{2G} \tag{13}$$

$$t_c = \frac{a}{c} \tag{14}$$

$$T \cong \frac{\hbar.c}{r_{BH}.k} \tag{15}$$

$$I = \sigma.T^4 \cong \frac{c.k^4 T^4}{(\hbar.c)^3} \tag{16}$$

where I is the intensity of radiation, k is the Boltzmann constant. Based on (16), relation (17) can be derived

$$k^4 \cong \sigma.\hbar^3.c^2 \tag{17}$$

Applying relations (13) - (17) to (12) again Hawking relation (7) is obtained in the meaning of limiting black hole in a given cosmological time.

4. Conclusions

· · The present contribution offers an independent derivation of Hawking formula concerning the quantum evaporation of black holes.

· · None of the arguments used contradicts the validity of the second law of thermodynamics.

· · A mutually consistent determination of the gravitational

field energy quantum, output and density, E_g, P_g and ϵ_g can be taken as an evidence on the correct localization and quantization of gravitational energy.

· · The calculations are of approximative nature when applying in the domain of strong fields.

References

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